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Geometric phase in entangled bipartite systems

H.T. Cui[a], L.C. Wang, and X.X. Yi[b]

Department of Physics, Dalian University of Technology, Dalian 116024, P.R. China

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Abstract. The geometric phase (GP) for bipartite systems in transverse external magnetic fields is investigated in this paper. Two different situations have been studied. We first consider two non-interacting particles. The results show that because of entanglement, the geometric phase is very different from that of the non-entangled case. When the initial state is a Werner state, the geometric phase is, in general, zero and moreover the singularity of the geometric phase may appear with a proper evolution time. We next study the geometric phase when intra-couplings appear and choose Werner states as the initial states to entail this discussion. The results show that unlike our first case, the absolute value of the GP is not zero, and attains its maximum when the rescaled coupling constant J is less than 1. The effect of inhomogeneity of the magnetic field is also discussed.

PACS. 03.65.Vf Phases: geometric; dynamic or topological – 03.65.Ud Entanglement and quantum nonlocality (e.g. EPR paradox, Bell's inequalities, GHZ states, etc.)

1 Introduction

A system can retain the information of its motion when it undergoes a cyclic evolution, in the form of a geometric phase (GP), which was first put forward by Pancharatnam in optics [1] and later studied explicitly by Berry in a general quantal system [2]. Great progress has been made in this novel region [3]. The original adiabatic condition in Berry's work has been removed by Aharonov and Anandan [4], and Samuel and Bhandari have generalized the geometric phase by extending to noncyclic evolution and sequential measurements [5]. At the same time a kinematic approach to the theory of the geometric phase has also been developed by Mukunda and Simon [6]. Recently the generalization to mixed states was conducted, first by Uhlmann in the mathematical context of purification [7], and then by Sjöqvist et al. based on the Mach-Zender interferometer [8]. Consequently the mixed-state geometric phase has been experimentally verified using both NMR interferometry [9] and single photon interferometry [10]. Recently the geometric phase for a mixed state was put forward by Singh et al. for the case of degenerate density operators [11] and a general formula for parallel transporting was also provided. Despite the great progress in this field, Bhandari recently raised the criticism that the definition for geometric phase in a mixed state fails when the interference fringes disappear [12]. This can be explained as the disappearance (appearance) of the geometric phase (off-diagonal geometric phase). The definition of off-diagonal geometric phase (OP) was first given by Manini et al. for pure states in adiabatic evolutions [13], and then was generalized to the non-adiabatic situation [14] and in mixed-states [15]. Moreover the off-diagonal geometric phase was studied in the degenerate case [16] and in bipartite systems [17]. Recently the effect of entanglement on the off-diagonal geometric phase has also discussed [18].

The quantum computation scheme for the geometric phase has been proposed based on the Abelian [19] or non-Abelian geometric phase [20], in which the geometric phase has been shown to be intrinsic against faults in the presence of some kind of external noise due to the geometric nature of the Berry phase. Consequently quantum gates based on the geometric phase have also been proposed in different systems [21], where the interactions play an important role for the realization of some specific operations.

Bipartite systems are of great importance in quantum computation, such as the transfer of quantum information, the construction of entanglement as well as the realizations of logic operations. Furthermore, it was found that GPs may be used to design quantum logic gates. These facts together give rise to the question of what are the geometric phase and its motion in bipartite systems. Recently some papers have addressed this issue [22,23], where the discussions respectively focus on the entanglement dependence of the geometric phase for subsystem and the coupling effect on the GP for subsystem under adiabatical evolution. However, another question remains open;

[a] e-mail: `cuiht@student.dlut.edu.cn`
[b] URL: `http://qpi.dlut.edu.cn`

how the entanglement or interaction affects the geometric phase for the whole system. This consideration is not trivial since any quantum information procession cannot be implemented by only one qubit. Moreover some interesting results may be found with the increment of the size of the system. In this paper, focusing on the entanglement and inter-subsystem couplings, we present explicit discussion of the geometric phases in bipartite systems.

For this purpose, our discussion is divided into several sections. In Section 2, we describe the model to be discussed in this paper and some formulas are also present for the calculation of the geometric phase. Then in Section 3 we study the geometric phase for two noninteracting particles and some interesting results can be found in this section. The intra-subsystem coupling effect on the geometric phase is studied in Section 4. Finally, conclusions are given at the end of this paper.

2 Model

In order to highlight the effect of the entanglement, we choose the initial state as

$$\rho(0) = \frac{1-r}{4} I + r|\Phi\rangle\langle\Phi|, \tag{1}$$

where $r \in (0, 1]$ determines the mixing of this state and I is the unit matrix in the 2×2 Hilbert space. Notice that for $r = 0$ equation (1) is the unit matrix and its geometric phase is trivial (always be zero), our discussion excludes this case. The state $|\Phi\rangle$ may be either of the following states,

$$\begin{aligned} |\varphi\rangle &= \sin\theta|11\rangle_{1,2} + \cos\theta|00\rangle_{1,2} \\ |\psi\rangle &= \sin\theta|10\rangle_{1,2} + \cos\theta|01\rangle_{1,2} \end{aligned} \tag{2}$$

where θ determines the degree of entanglement and $|1(0)\rangle_i$ $(i = 1, 2)$ is the eigenstate of Pauli operator σ_z. One should note that when $\theta = (3)\pi/4$, the equations above are Bell states and equation (1) are the Werner states [24], which plays an important role in quantum information processings, especially in quantum communication via noisy channels [25] and quantum distillation [26]. In general the mixed state (Eq. (1)), which was first introduced by Wootters [27], may be entangled; that can be judged by the Peres-Horodecki condition [28]. Equation (1) includes all possible cases, such as pure or mixed states and maximal or non-maximal entangled states. One should note that equation (1) is triplet-degenerate for $r \neq 1$.

We should point out that the initial state (Eq. (1)) is not a trivial generalizing from the pure case. The first term in equation (1) can be regarded as the noise, and the mixing coefficient r properly describes the intensity of noise. Recently the one-to-one correspondence between r of Werner state and the temperature T of the one-dimensional Heisenberg two-spin chain with a magnetic field B along the z-axis, has been established [29]. This connection give us the strong physical support for the initial state (Eq. (1)).

We choose the system composed of two spin-1/2 particles undergoing spin procession in an external time independent magnetic field in the z-direction. Then the Hamiltonian is

$$H = H_0 + H_I, \tag{3}$$

in which the free Hamiltonian H_0 and the XX interaction H_I are respectively

$$\begin{aligned} H_0 &= \omega_1 S_1^z + \omega_2 S_2^z \\ H_I &= g(S_1^{\dagger} S_2^- + S_1^- S_2^{\dagger}), \end{aligned} \tag{4}$$

where $g > 0$ is the antiferromagnetism coupling constant and S_i^z $(i = 1, 2)$ is the z component of spin operator respectively. $S_i^{\pm} = S_i^x + iS_i^y$ $(i = 1, 2)$ are the raising and decreasing operators of the z component of spin-1/2. Actually the Hamiltonian (Eq. (3)) is a two-qubit XX model, which is of fundamental importance to understand the relation between the entanglement and quantum correlation in interacting many-body systems. In general we suppose that ω_1 may not be equal to ω_2 because of the inhomogeneity of the external magnetic field. One will find in the following discussion that the inhomogeneity of the external magnetic field has a non-trivial effect on GP.

Some formulas should be addressed for the calculation of GP in this model. Recently GP for the mixed state has been discussed by Sjöqvist et al. in [8], based on the Mach-Zender interferometer and a formula was provided for calculation of GP for a mixed state,

$$\gamma_g = \mathrm{ArgTr}[U^{\parallel}(t)\rho(0)], \tag{5}$$

in which $U^{\parallel}(t)$ was defined as the parallel transportation operator. Based on this work, Singh et al. [11] studied GP for non-degenerate and degenerate mixed states and provided a general method for the construction of $U^{\parallel}(t)$ by imposing

$$U^{\parallel}(t) = U(t)V(t) \tag{6}$$

in which $U(t)$ is the unitary evolution operator and is equal to e^{-iHt} in our model and $V(t)$ is a blocked matrix, of which the elements are determined by

$$\begin{aligned} V_{\mu\nu} &= \langle\mu|e^{it\sum_{\mu',\nu'}\langle\mu'|H|\nu'\rangle|\mu'\rangle\langle\nu'|}|\nu\rangle, \\ &\quad |\mu\rangle, |\nu\rangle, |\mu'\rangle, |\nu'\rangle \in \{\text{degenerate subspace}\} \\ V_{kk} &= e^{i\langle k|H|k\rangle t}, |k\rangle \in \{\text{the remaining space}\}, \end{aligned} \tag{7}$$

where $|\mu\rangle, |\nu\rangle, |\mu'\rangle, |\nu'\rangle, |k\rangle$ are the eigenstates of $\rho(0)$, and the interference terms between the degenerate space and the other space are set to be zero in order to keep the parallel transporting in the degenerate space. One notes that for different mixed states one has different $U^{\parallel}(t)$. Based on these formulas, one can calculate GP. In the following calculations, we label the initial states as

$$\begin{aligned} \rho_1 &= \frac{1-r}{4} I_4 + r|\varphi\rangle\langle\varphi| \\ \rho_2 &= \frac{1-r}{4} I_4 + r|\psi\rangle\langle\psi|. \end{aligned} \tag{8}$$

and the geometric phases for these states are respectively calculated in the following parts.

$$\gamma_{g1} = \arctan \frac{-r\left(\sin\left[\cos 2\theta \frac{\omega_1+\omega_2}{2}t\right]\cos\frac{\omega_1+\omega_2}{2}t - \cos 2\theta \cos\left[\cos 2\theta\frac{\omega_1+\omega_2}{2}t\right]\sin\frac{\omega_1+\omega_2}{2}t\right)}{\frac{1-r}{2} + \frac{1+r}{2}\left(\cos\left[\cos 2\theta\frac{\omega_1+\omega_2}{2}t\right]\cos\frac{\omega_1+\omega_2}{2}t + \cos 2\theta \sin\left[\cos 2\theta\frac{\omega_1+\omega_2}{2}t\right]\sin\frac{\omega_1+\omega_2}{2}t\right)}$$

$$\gamma_{g2} = \arctan \frac{-r\left(\sin\left[\cos 2\theta \frac{\omega_1-\omega_2}{2}t\right]\cos\frac{\omega_1-\omega_2}{2}t - \cos 2\theta \cos\left[\cos 2\theta\frac{\omega_1-\omega_2}{2}t\right]\sin\frac{\omega_1-\omega_2}{2}t\right)}{\frac{1-r}{2} + \frac{1+r}{2}\left(\cos\left[\cos 2\theta\frac{\omega_1-\omega_2}{2}t\right]\cos\frac{\omega_1-\omega_2}{2}t + \cos 2\theta \sin\left[\cos 2\theta\frac{\omega_1-\omega_2}{2}t\right]\sin\frac{\omega_1-\omega_2}{2}t\right)} \quad (9)$$

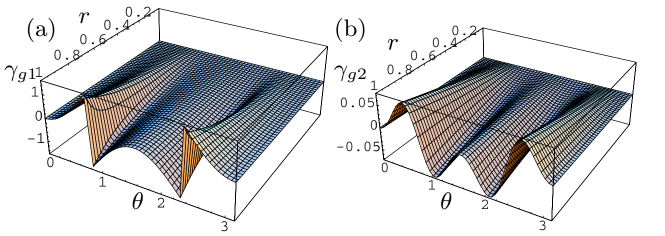

Fig. 1. (Color online) The geometric phases γ_{gj} $(j = 1, 2)$ [Arc] versus r, θ. For γ_{g1} (a), we have chosen $\omega_1 = \omega_2$ and $\omega_1 t = \pi/2$; whereas for γ_{g2} (b), $\omega_1 = 2\omega_2$ and $\omega_1 t = \pi$.

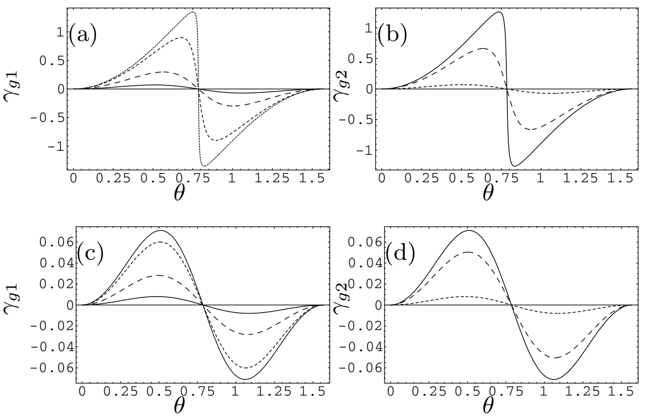

Fig. 2. The geometric phase [Arc] vs. the inhomogeneity of magnetic field $n = \omega_2/\omega_1$ with $r = 1$. Since the figures are symmetrical with the $\theta = \pi/2$, we only draw for $\theta \in [0, \pi/2]$. Part (a) corresponds to ρ_1 and the other parameters are the same as that of Figure 1a. The dotted, dashed, longer-dashed and solid lines correspond respectively to $n = 0.99, 0.9, 0.5, 0$. When $n = 1$, the geometric phase is not discontinued at the point $\theta = \pi/4$. Part (b) is for ρ_2 and the other parameters are the same as that of Figure 1b. The dotted, dashed and solid lines correspond to $n = 0.5, 0.1, 0.01$. When $n = 0$, the geometric phase is not discontinued in the point $\theta = \pi/4$. Parts (c) and (d) demonstrate the case that there is no the singularity (we have chosen $\omega_1 t = \pi/4$ for ρ_1 and $\omega_1 t = \pi/2$ for ρ_2 respectively).

3 g = 0 case

We first study the geometric phase without interaction. It should be pointed out that this situation still has interest in quantum information, such as quantum teleportation and communication in which the nonlocal correlation, i.e. the entanglement between two space-liked particles plays a crucial role. The geometric phases for ρ_i $(i = 1, 2)$ in this case can be easily obtained from the formulas in the former section,

see equations (9) above.

It is interesting to note that when the initial states are Werner states, the geometric phase is zero when the denominators in equations (9) are not vanishing simultaneously. Furthermore when $\omega_1 = \omega_2$, the geometric phase for ρ_2 is zero since ρ_2 is commutative with the Hamiltonian and it cannot pick up any geometric phase. A detailed demonstration for γ_{gj} $(j = 1, 2)$ with the parameters r, θ is shown in Figure 1. From the figures it is obvious that the absolute values of $\gamma_{g1(2)}$ attain the maximum when $\theta \neq \pi/4, 3\pi/4$ and because of the mixing, scaled by r, the absolute value of geometric phase is compressed and tends to be zero with $r \to 0$. We also note that a singularity about γ_{g1} appears when $\theta = (3)\pi/4$ and $r = 1$, as displayed in Figure 1a. At this point, the numerator and the denominator in the expression of γ_{g1} in equation (9) are simultaneously zero and the geometric phase is undefined. One has to calculate the so-called off-diagonal geometric phase to retain the information of the evolution [13]. Furthermore, our calculation shows that the singularity depends not only on the entanglement, but on how the system evolves.

With the consideration of the two noninteracting particles, it is of interest to discuss the effect of the inhomogeneity of the external fields. The results have been illustrated in Figure 2. It clearly shows that when there is a singular point, for ρ_1, the absolute value of the geometric phase with $n \to 1$ attains the maximum close to this point. Whereas, for ρ_2, this happens for $n \to 0$. We should emphasize that this phenomenon appears only when there exists the singularity, which is induced by the entanglement of the initial state. If there is no possibility of the appearance of the singularity, our calculation shows that the points where the absolute value of the geometric phase attains the maximum is independent of the inhomogeneity of the external field, shown as Figures 2c and 2d, and is always zero for the Werner state. Physically it is thus possible for this phenomenon to act as the signature of the singularity of the geometric phase, which usually originates from the degeneracy [13].

Another interesting consideration is that one supposes $\omega_2 = 0$, which corresponds to the case that particle A is processing with frequency ω_1 while particle B is kept dynamically fixed. In this case, the affect from the dynamics of the second particle is excluded and one can more

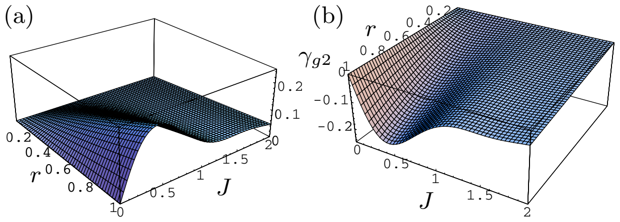

Fig. 3. (Color online) GP for γ_{g2} [Arc] versus r and rescaled coupling constant J. We have chosen $\theta = \pi/4$ (a) and $\theta = 3\pi/4$ (b) to highlight the coupling effect on the geometric phase, which corresponds to the Werner state as the initial state, and set $\omega_1 t = 2\omega_2 t = \pi$.

clearly check the effect of the entanglement on the geometric phase. The geometric phase for this special situation can be obtained by setting $\omega_2 = 0$ in equation (9). One finds that because of the entanglement, the geometric phase for particle 1 is also zero for the Werner state, and it is also possible for the singularity to appear, for example when $r = 1$ and $\omega_1 t = \pi$.

4 g $\neq$ 0 case

Because of the intra-subsystem coupling, the evolution of the system can be very different from the free case. So in this section we focus on the effect of coupling on the geometric phase. Obviously one notes that since $[H_I, \rho_1] = 0$, H_I has a trivial effect on γ_{g1} and thus this state has been excluded in this section. Based on the formulas in Section 2, GP are for ρ_2,

$$\gamma_{g2} = \arctan \frac{-r(\sin\lambda_1 t\cos\lambda_2 t - \frac{\lambda_2}{\lambda_1}\cos\lambda_1 t\sin\lambda_1 t)}{\frac{1-r}{2} + \frac{1+r}{2}(\cos\lambda_1 t\cos\lambda_2 t + \frac{\lambda_2}{\lambda_1}\sin\lambda_1 t\sin\lambda_1 t)} \tag{10}$$

in which

$$\lambda_1 = [(\omega_1 - \omega_2)\cos 2\theta - 2g\sin 2\theta]/2$$
$$\lambda_2 = \sqrt{(\omega_1 - \omega_2)^2 + 4g^2}/2.$$

The results are illustrated in Figure 3 with rescaled coupling constant $J = g/\omega_1$. Compared with the $g = 0$ case, we have chosen the Werner states ($\theta = (3)\pi/4$) as the initial states and $\omega_1 t = 2\omega_2 t = \pi$ with consideration of the inhomogeneity of the external magnetic field. Because of the coupling, the geometric phase for the Werner state is obviously very different from the free case. From these figures we also note that GP has a maximal or minimal value when the rescaled coupling constant is less than 1. Furthermore, when $J > 1$ the absolute value of GP decreases with increasing J. One also notes that because of the mixing, which is scaled by the parameter r, the absolute values of GP are compressed, which is same as the free case. The effect of the inhomogeneity of magnetic

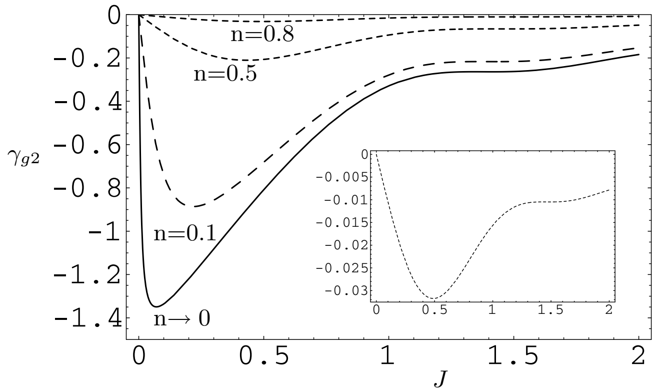

Fig. 4. GP [Arc] vs. the inhomogeneity of magnetic field $n = \omega_2/\omega_1$. We have chosen the Werner state ($\theta = \pi/4$) as the initial state with $r = 1, \omega_1 t = \pi$. The inset is for $n = 0.8$. Similar behavior can be found when $\theta = 3\pi/4$.

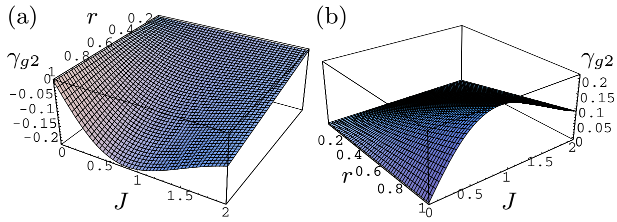

Fig. 5. (Color online) The geometric phase [Arc] for ρ_2 when $\omega_2 = 0$ versus r, J. Except of $\omega_2 = 0$, all other settings are the same as in Figure 3.

field are also discussed. Our calculation shows that with $\omega_2/\omega_1 \to 1$, the point that GP has a maximal or minimal value is infinitely close to $J = 0.5$ (Fig. 4).

We also investigate the GP when one particle is in zero field. This case is nontrivial since any system cannot avoid the affect from the other party. Also it can be used to manipulate the behavior of one particle by changing the coupling strength. To this end, we set $\omega_2 = 0$ and the geometric phase is illustrated in Figure 5. Different from Figure 3, the absolute value of GP attains the maximal value when $J \to 1$.

An interesting extension to the discussion above is in the limit of large $J = g/\omega_1$. In this case GP has a novel character, which is displayed in Figure 6. We note that GP tends to be zero in the limit of large J. This can be understood as that in the large limit of J the interaction H_I is dominant in equation (3)and the Hamiltonian is inclined to be commutative with the Werner state. Then the entanglement of the initial state tends to be unchanged.

Based on the analysis above, we can conclude that because of the inter-subsystem couplings, the geometric phase shows two different characters; in the weak coupling limit, the interaction obviously benefits the geometric phase, displayed in Figures 3 and 5. However, with further increment of the coupling, the GP decreases and tends to be zero infinitely. This phenomena can be explained easily only if one notes that in the infinite coupling limit, H_I is dominant and its eigenstates are just two of

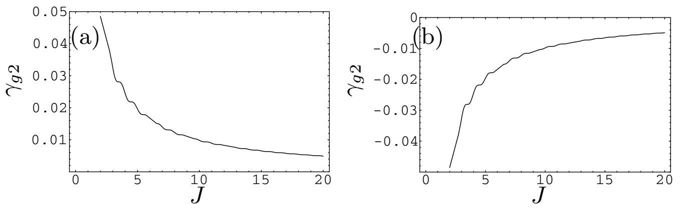

Fig. 6. γ_{g2} [Arc] in the limit of large J. The parameters have same values to that of Figure 3 and $r = 1$ is chosen.

the Bell states. The entanglement is destroyed in the weak coupling limit and then revived by the interaction in the infinite limit. The interesting relation between entanglement and interaction between two parties is an important aspect of quantum information.

5 conclusions

In conclusion, we have discussed the geometric phase for entangled mixed state equation (1) in an external magnetic field. For the free case ($g = 0$), our studies show that because of the entanglement, the geometric phase for the system displays two different types of character. The first is that if there was no singularity, the absolute value of the geometric phase attains a maximum when the initial state is not the Werner state, independent of the inhomogeneity of the external field (see Figs. 2c and 2d). Because of the mixing, scaled by r, it is compressed. Furthermore the geometric phase is always zero for the Werner state. The second behaviour occurs when a singularity appears, induced by the entanglement of the initial states under their proper evolution. In this case the geometric phase reaches a maximum with θ tending to the singular point (see Figs. 1a, 2a and 2b). We also discuss the situation that one particle is processing and another keeps dynamically fixed, and similar results can be found.

For the case $g \neq 0$, we choose the Werner states as the initial condition for facilitating our discussion. The results show that GP is completely determined by the rescaled coupling constant J and attains the maximal or minimal value when $J < 1$. Similar to the free case, we also discuss the effect of the inhomogeneity of the external magnetic field. The results show that for Werner state GP is always zero when system is in a homogeneous external magnetic field, independent of the interaction. Furthermore with $\omega_2/\omega_1 \to 1$, the absolute value of GP attains a maximum at the point $J \to 0.5$. The geometric phase for $\omega_2 = 0$ is also studied. Further study in the limit of large J displays a novel phenomena that GP tends to be zero and this result has been explained as the revival of entanglement in this limit.

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